

Examining the crossover from hadronic to partonic phase in QCD

Xu Mingmei, Yu Meiling, Liu Lianshou^{1,*}

¹*Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, China*

It is argued that, due to the existence of two vacua — perturbative and physical — in QCD, the mechanism for the crossover from hadronic to partonic phase is hard to construct. The challenge is: how to realize the transition between the two vacua during the gradual crossover of the two phases. A possible solution of this problem is proposed and a mechanism for crossover, consistent with the principle of QCD, is constructed. The essence of this mechanism is the appearance and growing up of a kind of grape-shape perturbative vacuum inside the physical one. A dynamical percolation model based on a simple dynamics for the delocalization of partons is constructed to exhibit this mechanism. The crossover from hadronic matter to sQGP as well as the transition from sQGP to wQGP in the increasing of temperature is successfully described by using this model with a temperature dependent parameter.

PACS numbers: 25.75.Nq, 12.38.Mh, 05.70.Fh

The theory of strong interaction — quantum chromodynamics QCD has a complicated phase structure [1]. It has been shown by lattice gauge theory [2] that at low temperature and density there is a confined, chiral symmetry breaking phase with hadrons as basic elements, while at high temperature and density is the deconfined, symmetry restoration phase with partons (quarks and gluons) as basic degree of freedom. It is found that at zero baryon density and high enough temperature, the transition from hadronic to partonic matter is of a crossover type [3].

Crossover is a gradual change of the system from one phase to the other without a definite transition point. The crossover between hadronic and partonic phases at high temperature has been firmly settled by lattice-QCD from thermodynamic argument, but really what happens in crossover; how does the system crossover from one phase to the other, are still open questions.

In this respect, it is worthwhile to consider a similar problem in QED — the Mott transition in atomic gas, where a neutral atom gas is transformed to electromagnetic plasma through the ionization of atoms one by one. However, such a mechanism is inapplicable to QCD, because the crossover of a hadron matter to quark-gluon plasma through the decomposition of hadrons to quarks (anti-quarks) one by one contradicts *color confinement*. According to color confinement, isolated color object cannot exist in physical vacuum. The energy of an isolated color object in physical vacuum is infinite [4].

Unfortunately, most of the current models on the market, which claimed crossover [5][6], have not taken this color confinement property of QCD into proper account. In these models the initial parton system evolves and hadronizes gradually. The hadronization condition is either that a parton ceases to interact with other partons [7] or that the total color interaction from a pair (or a three particle state) of quarks with the remaining

system vanishes [6]. In both cases the hadronization is carried out one by one. After a long time, the system in consideration is dominated by hadrons but there are still a few partons moving in the physical vacuum of hadronic matter [8], violating the confinement property of QCD.

The difficulty of crossover in QCD lies in the fact that QCD has different vacua — perturbative and physical — with partons and hadrons as basic element, respectively. How to realize the transition from one vacuum to the other during the gradual change — crossover — between partonic and hadronic phases is a big challenge [9]. In this letter we will discuss this problem and try to find a possible mechanism for the crossover in QCD compatible with the principle of color confinement.

For this purpose let us turn to another kind of model — the geometrical bond percolation model [10]. This model has no dynamical prescription but is nevertheless enlightening in constructing a crossover model for QCD [11, 12].

In the bond-percolation model, crossover is realized through the *formation of clusters*. In the lattice-version of the model, “a cluster is defined as a group of nearest-neighboring occupied sites that are linked by occupied bonds. The *cluster size* is the *number of sites* in a cluster” [10]. In our case, the sites are distributed continuously in the space and will be in the following referred to as *cells*. The cluster-formation could be understood as: originally, isolated cells are color singlet hadrons, when they are connected by bonds to form clusters, color can flow among them through bonds, and only the cluster as a whole keeps to be color singlet.

Clusters could be of various sizes. The formation of an *infinite cluster*, which is defined in a finite system as a cluster extending from one boundary to the other, *i.e.* a cluster having, at least in one dimension, the system size, is taken as the appearance of a new constituent in the system. At that time there are still a lot of clusters of various-sizes, cf. Fig. 1(a), *i.e.* the system as a whole has not yet turned to the new phase. So we consider the appearance of an infinite cluster as the *starting point of the crossover* of the system to a new phase. The *crossover process is completed when all the cells in the*

*Electronic address: liuls@iopp.ccnu.edu.cn

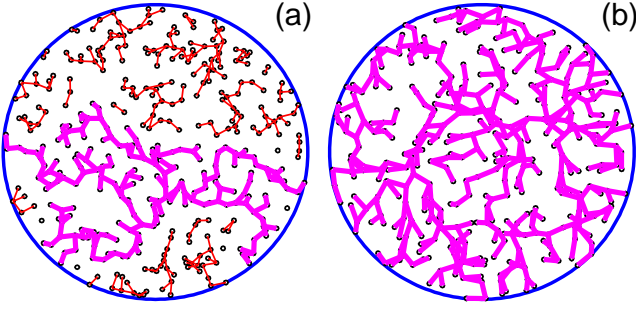


FIG. 1: (Color online) Continuously distributed cells connected by bonds form clusters. In (a) the big cluster extending from the left- to the right-boundary is an *infinite cluster*. In (b) all the cells are connected to an infinite cluster.

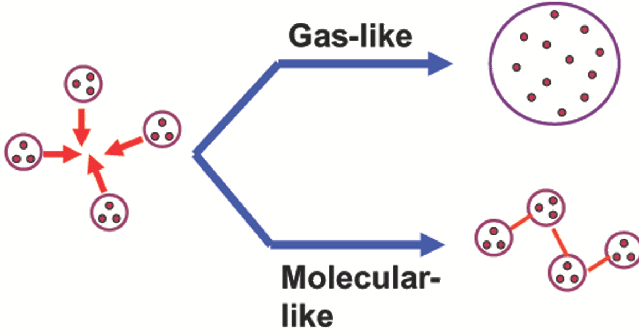


FIG. 2: (Color online) Two possible forms of nucleon (or hadron) aggregation: gas-like and molecular-like. In both cases only the whole aggregation is color singlet.

system are connected to a unique cluster, cf. Fig. 1(b). In this process no isolated color object will appear in physical vacuum, and thus no contradiction with QCD.

Inspired by this achievement we propose our basic assumption as the following. *The aggregation of hadrons could be of two forms: gas-like or molecular-like, cf. Fig. 2 [13].* In the first case all the hadrons in an aggregation are melted together, forming a big “bag” with quarks and gluons moving inside, which is usually referred to as *QGP droplet*. In the second case, the hadrons (or cells) in a cluster, like the atoms in a molecular, in spite of being no longer color singlets, still keep their individuality, *i.e.* still occupy separate space regions, connected with each other by bonds, forming a color singlet cluster.

The cells, *i.e.* the separated space regions occupied by the colored parton-groups, are essentially “holes” dug by the groups of colored parton in physical vacuum [4]. These holes can be viewed as potential wells if we add energy as an extra ordinate. Partons can tunnel through the potential barriers between neighboring wells all over the color-singlet cluster, which is referred to as *delocalization* [14].

When the crossover process is completed, all the wells in the system are connected to a unique cluster, forming a

grape shape perturbative vacuum [15] of the nuclear size in 3-D, cf. Fig.1(b). Meanwhile, there are still isolated bubbles of physical vacuum in the system [12], cf. the white regions in Fig.1(b). The groups of colored parton are confined in the separate wells, or cells, inside the *grape shape* perturbative vacuum, being able to exchange color among themselves via quantum tunneling through the potential barriers.

It could be expected that, the physical-vacuum bubbles can not survive long in the environment of the grape-shape perturbative vacuum and will soon be transformed to perturbative too. The groups of colored parton can then move around in the whole system, resulting in a quark gluon matter possessing the property of *perfect fluid*, which is conventionally referred to as *strongly coupled QGP*, sQGP [16, 17].

At still higher temperature the potential barriers between neighboring wells will drop to zero and all the wells disappear. In this case all the parton-groups will be disassembled to independent partons and the sQGP turned to *weakly coupled QGP*, wQGP.

Let us try to construct a simple model for realizing the above assumptions and arguments. Our aim is to answer the question: *how to crossover in QCD*. For simplicity, we take the initial system to be a nucleon gas.

The model Hamiltonian for the 6 quarks in two near-by cells is assumed to be [14]

$$\mathcal{H} = \sum_{i=1}^6 \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{\text{cm}} + \sum_{i < j} V_{ij}^C, \quad (1)$$

where T_{cm} is the center-of-mass kinetic energy. V_{ij}^C is the color interaction, which will be chosen as a square-confinement potential $V_{ij}^C = -a_c \vec{\lambda}_i \cdot \vec{\lambda}_j r_{ij}^2$ when the quarks i, j belong to one and the same cell. When they belong to two nearby cells, the infinite potential in between will drop down, forming a potential barrier, and a parametrization $V_{ij}^C = -a_c \vec{\lambda}_i \cdot \vec{\lambda}_j \frac{1 - e^{-\mu r_{ij}^2}}{\mu}$ will be used, where μ is a model parameter. It turns out that as the increasing of μ the maximum distance S_0 for delocalization increases, cf. Fig. 3 below. Since at higher temperature the quarks will be more free to move, *i.e.* S_0 will be larger, we require μ to be an ascending function of T . From dimensional consideration we assume $\mu \propto T^2$.

In doing variational calculation the trial wave function of the two-cell system in adiabatic approximation is chosen to be an antisymmetric six-quark product state [14]

$$|\Psi_6(S)\rangle = \mathcal{A} \left[\prod_{i=1}^3 \psi_L(\mathbf{r}_i) \prod_{i=4}^6 \psi_R(\mathbf{r}_i) \eta_{I_1 S_1}^{B_1} \eta_{I_2 S_2}^{B_2} \chi_c^{B_1} \chi_c^{B_2} \right]_{00}^{(2)}, \quad (2)$$

where \mathcal{A} is the anti-symmetrization operator, which permutes quarks between the two cells; $[\dots]_{00}$ means that the spin, isospin and color of the two cells are coupled to a particular color singlet state with total spin and isospin equal zero. For the orbital motion, we have the left (right) single-quark orbital wave function

$\phi_L(\mathbf{r}_i) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{(r_i - \frac{S}{2})^2}{2b^2}}$, $\phi_R(\mathbf{r}_i) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{(r_i + \frac{S}{2})^2}{2b^2}}$, where $\pm \frac{S}{2}$ are the cell-centers, S is the distance between the two cells. b is a baryon-size parameter. Delocalized orbit is defined as $\psi_L(\mathbf{r}) = \frac{1}{N}[\phi_L + \epsilon\phi_R]$, $\psi_R(\mathbf{r}) = \frac{1}{N}[\epsilon\phi_L + \phi_R]$, where ϵ is a variational parameter characterizing the degree of delocalization and N a normalization factor. At each separation S , ϵ is determined by minimizing the energy $E(S) = \frac{\langle \Psi_6(S) | \mathcal{H} | \Psi_6(S) \rangle}{\langle \Psi_6(S) | \Psi_6(S) \rangle}$. The model parameters are: $m_u = m_d = 313$ MeV, $b = 0.603$ fm, $a_c = 101.14$ MeV/fm², while μ is leaving as a free parameter.

It turns out that the model gives a reasonable amount of delocalization. When the two cells are close together, *i.e.* S is small, the delocalization is large $\epsilon=1$, but when the two cells separate and S increases to a certain distance S_0 , the delocalization degree ϵ suddenly drops to zero. The dependence of S_0 on parameter μ is shown in Fig. 3.

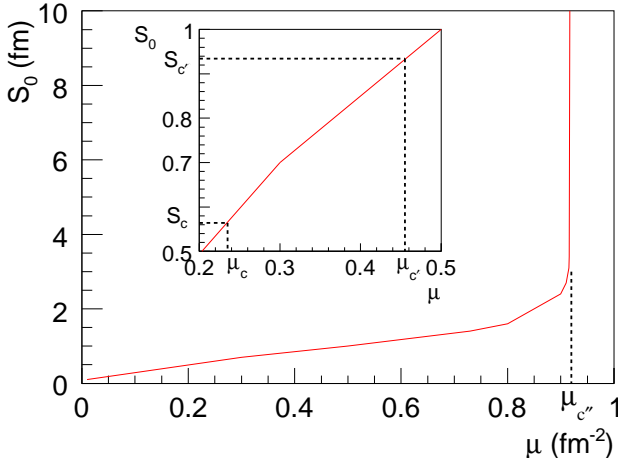


FIG. 3: The μ dependence of S_0 determined by the dynamics of our model. Small pad shows a local part which is useful in determining the critical values of parameter μ .

S_0 provides us a distance within which the quarks have equal probability to be simultaneously in the two cells, *i.e.* within which bond is likely to be formed. We will use it to construct a bond-percolation model to realize the crossover in QCD. For simplicity, we will consider the system to be a 2 dimensional static one.

The initial configuration consists of 2×197 cells, which are small spheres (circles in 2-D) of hard-core radius $r_e = 0.1$ fm distributed randomly in a big sphere (circle in 2-D) of radius $R = 7$ fm. A cell with center departing from the center of the big sphere (circle) farther than $R - r_e$, is considered as a *boundary cell*.

The percolation procedure is as follows:

- 1) Randomly select a cell α as a *mother cell*.
- 2) Define the cells with $|\mathbf{r} - \mathbf{r}_\alpha| \leq S_0$ as *bond-candidate cells*, randomly select 3 of them to form bonds connected to the mother cell α , and call the latter as *daughters*.

If the number of candidate-cells is less than 3, then the number of daughters is equal to the candidate number.

- 3) For every daughter of cell α randomly select 2 bond-candidate cells of it to form bonds, and call them granddaughters. Repeat the procedure until no bond-candidate cell can be found anymore.

- 4) Then choose another cell β from the left unbounded cells as another mother cell, and repeat the procedure starting from step 2.

- 5) Repeat step 4 until all the cells are exhausted.

In this way, every cell is assigned to a cluster. In each cluster, find the boundary cells if any, and calculate the distance between every two boundary cells. Denote the maximum distance by d . A cluster with $d > \sqrt{2}R$ is taken as an *infinite cluster*.

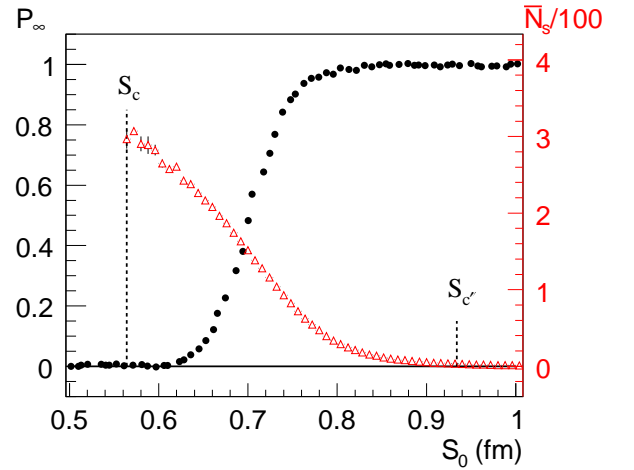


FIG. 4: (Color online) Numerical results P_∞ (full circles) and $\bar{N}_s/100$ (open triangles) versus S_0 for 10 000 events. The dashed lines correspond to the 2 thresholds, *i.e.* T_c the place where P_∞ starts to increase from zero, $T_{c'}$ where the mean number \bar{N}_s of cells outside an infinite cluster tends to zero.

A configuration with all the cells assigned to clusters is called an *event*. Generate N events, and determine in each event whether there are infinite cluster(s). Suppose M is the number of events with infinite cluster(s), the *probability for the appearance of event with infinite-cluster(s)* can be expressed as $P_\infty = \lim_{N \rightarrow \infty} M/N$. The dependence of P_∞ on S_0 is plotted in Fig. 4 as full circles. We see that at a certain value $S_0 = S_c$, P_∞ starts to increase from zero, and the system starts to crossover to a new phase. The corresponding value of μ is $\mu = \mu_c$.

Let N_s be the number of cells outside of an infinite cluster in an event. The mean \bar{N}_s versus S_0 is plotted in Fig. 4 as open triangles. The point where $\bar{N}_s = 0$ marks the accomplishment of crossover and will be denoted by $S_0 = S_{c'}$. The corresponding value of μ is $\mu_{c'}$, *cf.* the dashed lines in Fig. 4 and in the small pad of Fig. 3.

The starting and ending points μ_c and $\mu_{c'}$ of crossover can be extracted from Fig's. 3, 4. Assuming $\mu \propto T^2$ we get the corresponding temperatures: $T_{c'} \approx 1.39 T_c$.

It is interesting to see from Fig. 3 that as μ increases further from $\mu_{c'}$, at a certain point $\mu_{c''}$, S_0 increases sharply to infinity. An infinite S_0 means that partons can flow from one well to the other no matter how far away they are. This means that at this point the potential barriers have dropped to zero, and there is no well in the vacuum anymore. Partons can move freely inside the whole system, and the sQGP turns to wQGP. Extracting the point $\mu_{c''}$ for this transition from the figure and assuming again $\mu \propto T^2$, we get $T_{c''} \approx 1.98T_c$ [18].

In this letter a possible mechanism for the crossover in QCD, compatible with the principle of color confinement, is proposed. The way for vacuum-change during the gradual crossover between the two phases is via the following steps.

1) As the increasing of temperature, partons in hadrons start to be delocalized, tunneling between neighboring hadrons, and change the latter to *colored parton-groups* located in potential wells.

2) The wells connected by tunnels form color singlet clusters, which grow up in the physical vacuum, and eventually become a big cluster of the system size. Inside this cluster is a grape-shape perturbative vacuum, while outside of it is physical vacuum bubbles, which will change to perturbative as the further increase of temperature, and the colored parton-groups in the wells can then move around, forming a fluid-like matter — sQGP.

3) The potential barriers will drop to zero at a higher temperature and all the wells disappear, the sQGP is

then turned to wQGP.

Last but not least, a dynamical percolation model is constructed for the first time, which is superior to the purely geometrical percolation models. 1) Our model has dynamical foundation and the cells and bonds have definite physical meaning. 2) We use a maximum length S_0 for bond-formation, obtained from dynamical calculation, and construct 3 bonds *randomly* within this length. This is to substitute the bond-formation probability p put in the geometrical models by hand. 3) Our model has a temperature-dependent parameter μ and is able to deal with the crossover as well as the transition from sQGP to wQGP in the increasing of temperature, while the geometrical percolation models are able to treat only the effect of the increasing of density.

In the proposed scenario the crossover is accomplished within a temperature range from T_c to $T_{c'}$. It could be expected that at some energies lower than 200 GeV, the produced dense matter may be at a mediate stage of crossover, which in our scenario consists of an infinite cluster accompanied by a large number of other clusters, cf. Fig. 1(a). To study the properties of such a structure of matter theoretically and experimentally is worthwhile, which will deepen our understanding on QCD phase diagram and critical dynamics.

We thank Wang Fan, Ping J. L., Li J. R., Wu Y. F., Zhuang P. F., H. C. Ren, J. Shock, N. Xu and H. Z. Huang for helpful discussions. This work is supported by NSFC under projects-10775056, 90503001.

-
- [1] T.D. Lee, G.C. Wick, Phys. Rev. D **9**(1974)2291; T.D. Lee, Rev. Mod. Phys. **47** (1975) 267; E.V. Shuryak, Phys. Rept. **61** (1980) 71; for a recent review see e.g. T. Schäfer, lecture given at HUGS 2005, hep-ph/0509068.
 - [2] F. Karsch, E. Laermann and A. Peikert, Phys. Lett. B **478**(2000)447; F. Karsch, Lecture Notes in Physics **583** (2002) 209; Z. Fodor and S.D. Katz, JHEP **0404** (2004) 050.
 - [3] Y. Aoki et al., Nature **443**, 675-678 (2006).
 - [4] See e.g. T. D. Lee, Particle physics and introduction to field theory, Harwood academic publishers, Chur, London, Newyork, 1981, p.397 – 401.
 - [5] C. M. Ko, talk given at the 4th Intern. Workshop on Critical Point and onset of Deconfinement, Darmstadt. July 2007.
 - [6] M. Hofmann, M. Bleicher, S. Scherer, L. Neise, H. Stöcker and W. Greiner, Phys. Lett. B **478** (2000) 161.
 - [7] Z.W. Lin, C.M. Ko, B.A. Li, B. Zhang and Subrata Pal, Phys. Rev. **C72**(2005) 064901.
 - [8] See, e.g., Fig. 2 in Yu Meiling, Du Jiaxin and Liu Lian-shou, Phys. Rev. **C74** (2006) 044906 .
 - [9] There are some models where a fast hadronization is discussed with color confinement explicitly taken into account, see e.g. T. S. Biro, P. Levai, J. Zimanyi, Phys. Rev. C **59**, 1574-1584 (1999). However, the transition between different vacua in QCD has not been studied there.
 - [10] K. Christensen and N. R. Moloney, Complexity and Criticality, Imperial College Press, 2005, p.79.; M. B. Isichenko, Rev. Mod. Phys. **64** (1992) 961.
 - [11] The idea of a connection between deconfinement and percolation is quite old. The pioneer works are G. Baym, Physica **96A** (1979) 131 and the references cited in [12].
 - [12] H. Satz, Nucl. Phys. A **642** (1998) 130c; Int. J. of Mod. Phys. **A21** (2006) 672.
 - [13] In the paper T. Celik, F. Karsch and H. Satz, Phys. Lett. **97B** (1980) 128, two alternatives of connectivity had been suggested in a somewhat different form.
 - [14] F. Wang, G.H. Wu, L.J. Teng and T. Goldman, Phys. Rev. Lett. **69** (1992) 2901.
 - [15] There is non-perturbative barrier-tunnelling in the grape-shape vacuum, but except that it is perturbative.
 - [16] I. Arsene et al., BRAHMS Collaboration, Nucl. Phys. **A757**, 1-27 (2005); K. Adcox et al., PHENIX Collaboration, Nucl. Phys. **A757**, 184-283 (2005); B.B. Back et al., PHOBOS Collaboration, Nucl. Phys. **A757**, 28-101 (2005); J. Adams et al., STAR Collaboration, Nucl. Phys. **A757**, 102-183 (2005)
 - [17] G. E. Brown, B. A. Gelman and Mannque Rho, Phys. Rev. Lett. **96**, 132301 (2006); E. Shuryak, Nucl. Phys. A **774**, 387 (2006).
 - [18] There are analytical calculations from QCD-based models for the thresholds. The results are consistent with ours. See e.g. Lianyi He, Meng Jin and Pengfei Zhuang, hep-ph/0511300, and the papers cited therein.